PHYSICAL REVIEW E **78**, 056301 (2008)

Noise influence on pole solutions of the Sivashinsky equation for planar and outward propagating flames

R. V. Fursenko,[1] K. L. Pan,[1,*] and S. S. Minaev[2]

[1]*Department of Mechanical Engineering, National Taiwan University, Taipei 10617, Taiwan, Republic of China*

[2]*Khristianovich Institute of Theoretical and Applied Mechanics, Russian Academy of Sciences, Novosibirsk 630090, Russia*

(Received 11 June 2008; revised manuscript received 5 September 2008; published 10 November 2008)

The dynamics of planar and outward propagating cylindrical flames has been studied in terms of exact solutions of the Sivashinsky equation with a random force term. The force term models the computational roundoff errors or a variety of perturbations of physical origins. In contrast to noiseless conditions, the number of poles in the system does not conserve and new poles appear due to the external forcing. It was found that modification of the pole solutions taking into account the appearance of new poles captures the features typical for the hydrodynamically unstable flames, which cannot be detected by the pole solutions with a fixed number of poles. Investigations based on the pole solutions make it possible to exclude the uncontrolled numerical noise that is always present in direct computations of the Sivashinsky equation, and to examine the interplay between noises and hydrodynamic instability. The study clearly demonstrates that the presence of noises is a necessary condition for flame acceleration.

DOI: 10.1103/PhysRevE.78.056301 PACS number(s): 47.70.Pq, 47.20.Ky

I. INTRODUCTION

A spherical flame expanding out from the ignition source is one of the most basic configurations of premixed combustion. It was observed in experiment [1] that the front of an outward propagating flame does not remain smooth given enough time. Instead, a self-similar structure of cells is formed on the flame surface and the flame front undergoes a noticeable acceleration. The cellularization of flame fronts was associated with the intrinsic long-wave combustion instability, also known as the Darrieus-Landau or the hydrodynamic instability [2,3].

To describe an outward propagating flame, Sivashinsky [4,5] proposed a weakly nonlinear integro-differential equation of flame front that is motivated by the physics and captures a number of essential features typical for the flames that are driven by hydrodynamic instability. Though the reduced equation is valid only for the small-heat-release limit (i.e., for the case of the vicinity of the density ratio between unburned and burned gases, $E=\rho_u/\rho_b$, to unit), it generates qualitatively reasonable solutions even when the thermal expansion coefficient E is extrapolated over realistic values [6]. Numerous works [5,7,8] dedicated to numerical simulations of expanding flames governed by the Sivashinsky equation clearly indicate front cellularization and substantial flame acceleration. It was proposed that these phenomena resulted from explicit and/or implicit forcing, which is always presented both in experiments and calculations. The effect of forcing has been studied in [7] by computation of the Sivashinsky equation with an additional term that describes external noise. A clear correlation between the strength of the forcing and the flame expansion rate was proposed. However, numerical simulations of the Sivashinsky equation subjected to external white noise may provide only circumstantial support of the conjecture of noise-driven acceleration due to the extreme sensitivity of the equation to uncontrolled numerical noises that are always presented in the calculations. The attempt to exclude the effects of roundoff errors was undertaken in [8] by using some filter. Yet these authors noted that even filtered numerical integrations of the Sivashinsky equation were not noise-free and the remaining computational errors still affected the flame front dynamics.

The Sivashinsky equation allows for a whole class of exact pole solutions [9–11], which have proved to be extremely useful for understanding the wrinkled flame dynamics. Analytical technique based on pole decomposition provides a set of ordinary differential equations for the positions of the poles in the complex plane that determine the geometry of the developing front. The pole description gives an exact representation of the flame dynamics without noise. Nonsusceptibility of the exact solutions to the numerical noises and roundoff errors presents the way to add totally controlled noise in the system so as to clarify its influence on flame dynamics.

For the noiseless condition, the pole dynamics always conserves the number of poles given by the initial condition. As a result, the exact solution of the Sivashinsky equation is incapable of describing the continuing self-similar cellularization of the expanding front and the flame self-acceleration that were observed in experiments and numerical simulations [8,12]. Such a sharp contradiction between the analytical result and the numerical simulation of the Sivashinsky equation was also revealed in the case of wrinkled flames propagating in the duct. According to the theoretical prediction [9,10], the nondimensional velocity of propagation of the wrinkled flame is bounded by the value $1+(E-1)^2/8$, where E is the thermal expansion coefficient. Furthermore, numerical simulations [7,13] predict a significantly larger value of the flame velocity, which increases with the flame size (i.e., characteristic diameter of the duct) L. It is therefore tempting to conjecture that noise may play an important role in affecting the dynamical behavior of the flame fronts driven by the hydrodynamic instability.

*Author to whom correspondence should be adressed; panpeter@ntu.edu.tw

 ©2008 The American Physical Society

In terms of the pole decomposition technique, the effect of noise may be manifested by the appearance of new poles in the system. The theoretical estimation of the influence of an additional pole on the flame dynamics has been presented in [14,15]. It was shown that the addition of a new pole may lead to the formation of a new secondary cusp on the outward propagating flame front [15]. It reveals the possibility of linking the self-similar structure and the pole solutions with controlled noises. In spite of this, the effects of external random forces on the exact solutions of the Sivashinsky equation were still not clearly understood.

The main goal of this work is to investigate the influence of noise on the characteristics of a flame propagating in the duct and outwardly expanding flames in terms of pole dynamics. Calculations based on the exact solutions make it possible to exclude the effect of uncontrolled numerical noise and to examine the interplay between noises and instability. The study shall demonstrate clearly that the presence of noises is a necessary condition for flame acceleration. A qualitative comparison between the calculations of pole solutions and numerical simulations of the Sivashinsky equation is also discussed.

From the physical point of view, the Sivashinsky equation with random force terms may describe the flames propagating through the space with localized nonuniformities or flow disturbances (such as dust or vortices). Therefore, these issues are relevant also to the problems of combustion in the particle-laden flows.

II. MATHEMATICAL MODEL

A simplified mathematical model for cellularization of hydrodynamically unstable flames was obtained in [4,5], and in this work, we have performed an investigation for the evolution of unstable flame fronts governed by the Sivashinsky equation. After appropriate rescaling, the Sivashinsky equation describing the nonlinear evolution of outward propagating cylindrical flames can be written as [5]

$$\frac{\partial R}{\partial t}-\frac{\gamma}{2R_f(t)}\hat{I}(R) = \frac{1}{R_f^2(t)}\frac{\partial^2 R}{\partial \varphi^2}+\frac{1}{2}\left(\frac{1}{R_f(t)}\frac{\partial R}{\partial \varphi}\right)^2+1+f(\varphi,t), \quad 0<\varphi<2\pi. \tag{1}$$

Here $r=R(\varphi,t)=R_f(t)+h(\varphi,t)$ is the shape of a perturbed cylindrical flame front expressed in dimensionless variables constructed based on the Markstein length Ma; $R_f(t)=(1/2\pi)\int_0^{2\pi}R(\varphi,t)d\varphi$ is the mean radius of the expanding flame; t is nondimensional time, in units of Ma/U_b; $f(\varphi,t)$ is an upstream perturbation of the unburned gas velocity field [7]; $\gamma=E-1$, where $E=\rho_u/\rho_b$ is the thermal expansion coefficient defined as the ratio between the densities of unburned and burned gases, respectively. The operator $\hat{I}(R)$ is defined in [5] as

$$\hat{I}(R)=\frac{1}{\pi}\sum_{n=1}^{\infty} n\int_0^{l}\cos[n(\varphi-\varphi^*)]R(\varphi^*,t)d\varphi^*.$$

In the absence of external force, $f(\varphi,t)=0$, the solution of the Sivashinsky equation can be expanded in functions that depend on N poles whose position $z_j=x_j+iy_j$, $j=1,\ldots,N$, in the complex plane is time-dependent [9,12,16–19],

$$u(\varphi,t)=\sum_{k=1}^{N}\cot\left(\frac{\varphi-z_k}{2}\right)+\text{c.c.}, \tag{2}$$

$$R(\varphi,t)=2\sum_{k=1}^{N}\ln[\cosh(y_k)-\cos(\varphi-x_k)]+C(t), \tag{3}$$

where $u\equiv\partial R/\partial t$ and $C(t)$ is a function of time. The function (3) is a superposition of quasicusps (i.e., cusps that are rounded at the tip). The real part of the pole position (i.e., x_j) describes the angular coordinate of the maximum of the quasicusp, and the imaginary part of the pole position (i.e., y_j) is related to the depth of the quasicusp. As y_j decreases (increases), the height of the cusp increases (decreases). The variables x_j and y_j determine also the locations of virtual sinks in the combustion product for the velocity potential of fresh gas and the positions of virtual sources in the fresh-gas region for the velocity potential of combustion product [12]. Therefore, the data about variables x_j and y_j make it possible to reconstruct the velocity fields on both sides of the front at any time. The positions of the poles satisfy the system of ordinary differential equations [9,15],

$$-R_f^2\frac{dz_n}{dt}=\cot\left(\frac{z_n-\bar{z}_n}{2}\right)+\sum_{k=1,k\neq n}^{N}\left[\cot\left(\frac{z_n-z_k}{2}\right)+\cot\left(\frac{z_n-\bar{z}_k}{2}\right)\right]+i\frac{\gamma R_f}{2}\,\mathrm{sgn}[\mathrm{Im}(z_n)]$$

$$\frac{dR_f}{dt}=1+2\sum_{k=1}^{N}\frac{dy_k}{dt}+2\left(\frac{\gamma}{2}\frac{N}{R_f}-\frac{N^2}{R_f^2}\right). \tag{4}$$

In noiseless conditions $[f(\varphi,t)=0]$, the total number N of poles is conserved in time and equal to the number of poles that exist in the initial conditions. Linear stability analysis [9] has showed that the exact solutions (2)–(4) with a constant number of poles are stable with respect to small perturbations. Thus, the pole solutions of Sivashinsky equation (1) with zero force term $[f(\varphi,t)=0]$ are unsusceptible to the influence of numerical noise due to the exponential damping of perturbations generated by roundoff errors. In contrast, the direct simulation of the Sivashinsky equation gives no way to isolate the noise effect because of extreme sensitivity of this equation to the uncontrolled numerical noise always present in the calculations [7,15,20].

A random pointwise set of perturbations uniformly distributed in time and in space is a suitable model for both the computational roundoff errors and a variety of perturbations of physical origins [7]. Such a model can be written as

$$f(\varphi,t) = \sum_{m=1}^{M} g_m(\varphi)\delta(t-t_m). \tag{5}$$

Let us show that the impulselike noise (5) to be inserted in Eq. (1) can be modeled in terms of the pole dynamics as the appearance of new poles in the system (4). As demonstrated in [12,14], any function that can be represented in Fourier series can be approximated to a desired accuracy by pole decomposition, Eq. (2). Therefore, we can assume without loss of generality that the function $g_m(\varphi)$ is represented as finite sums,

$$g_m(\varphi) = 2\sum_{k=1}^{N_{g,m}} \ln[\cosh(y_k) - \cos(\varphi - x_k)],$$

$$\frac{\partial g_m(\varphi)}{\partial \varphi} = \sum_{k=1}^{N_{g,m}} \cot\left(\frac{\varphi - z_k}{2}\right) + \text{c.c.}$$

In this case, the solution $u(\varphi,t)$ of Eq. (1) with a force term, Eq. (5), can be expanded in the series,

$$\frac{\partial R(\varphi,t)}{\partial \varphi} = u(\varphi,t) = \sum_{k=1}^{N} \cot\left(\frac{\varphi - z_k}{2}\right) + \sum_{m=1}^{M} H(t-t_m) \sum_{k_m=1}^{N_{g,m}} \cot\left(\frac{\varphi - z_{k,m}}{2}\right) + \text{c.c.}, \tag{6}$$

where $H(x)$ is the Heaviside function. By substituting Eq. (6) in Eq. (1), which is differentiated on φ, and using the symbolic equality $(\partial/\partial t)H(t-t_m)=\delta(t-t_m)$, we obtain the system of ordinary differential equations for positions of poles in the same manner as the system of Eq. (4) was obtained. This system is identical to that of Eq. (4) except that the number of poles is changed in time, i.e., the $N_{g,m}$ poles are added in the system at the time moments $t=t_m$, $m=1,\ldots,M$. Thus, the exact solutions of the Sivashinsky equation enable us to exclude the effect of uncontrolled roundoff errors and consider only the external noise described by the forcing term of Eq. (5).

III. RESULTS AND DISCUSSION

A. Flame in the planar duct

All derivations of the previous section can be extended to the case of flame propagating in the planar duct with transverse size L. A detailed description of the Sivashinsky equation and pole decomposition technique for this case has been given in [4,9,14]. Under the noiseless condition $[f(\varphi,t)=0]$, there is only one stable stationary solution for the disturbed flame front, which is geometrically represented by a giant cusp and analytically by $N_c=\gamma L/8\pi$ poles that are aligned on one line parallel to the imaginary axis [9]. It can be shown that for any number of poles in the initial condition, this is the only attractor of the pole dynamics [9,14]. Calculations with different initial conditions containing a large number of poles demonstrate that after a short transition stage this steady state is established. As shown analytically in [10], the velocity of the stationary propagating flame is independent of the size of the duct L and bounded by the value $V_c=1+\gamma^2/8$. Contrary to the theoretical results, numerical simulations [7,13] of the Sivashinsky equation show that the flame front velocity increases with the growth of the flame size and can considerably exceed the predicted maximal velocity V_c. It is conceivable that this contradiction between the analytical and numerical results would be explained by the influence of numerical noise.

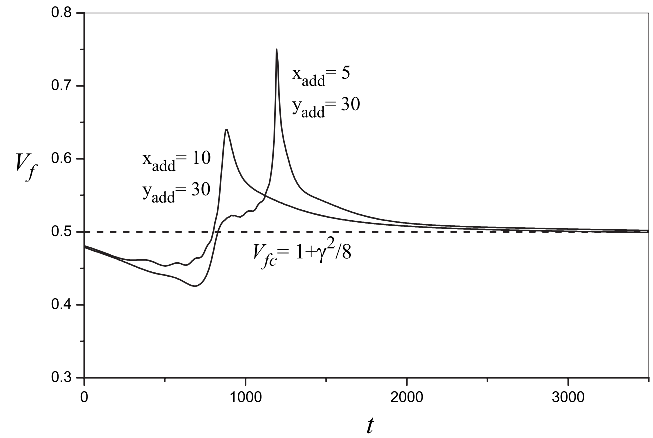

FIG. 1. Time dependency of the velocity of the flame propagating in the duct with $L=90$ and $\gamma=2$, disturbed by adding a pole at the position $(x_{\text{add}}, y_{\text{add}})$.

Figure 1 shows the temporal dependency of the front velocity of the steady propagating planar flame subjected at the initial time moment to the impulselike noise (5) with $M=1$ and $N_{g,1}=1$, which is modeled by the appearance of only one new pole. As illustrated in Fig. 2, the small perturbation of the flame surface grows rapidly and forms a new cusp that moves toward the giant cusp and subsequently merges with

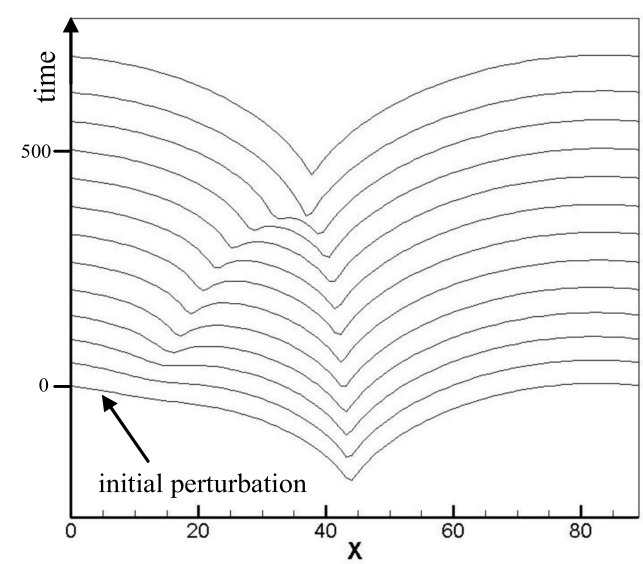

FIG. 2. Evolution of a flame upwardly propagating in the duct with $L=90$ and $\gamma=2$. The steadily propagating flame was perturbed by addition of one new pole at the position $x_{\text{add}}=5$ and $y_{\text{add}}=30$.

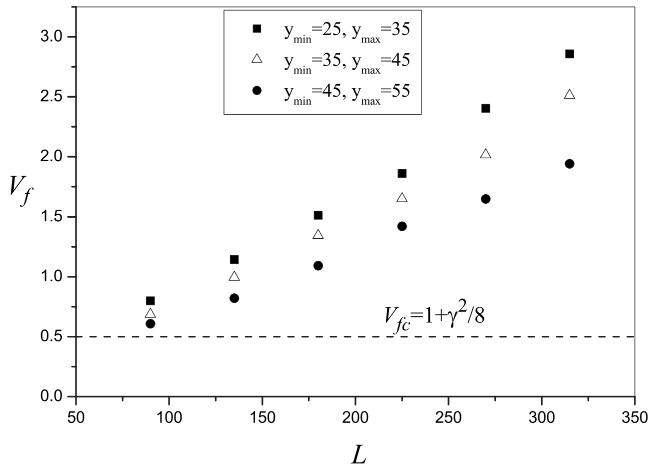

FIG. 3. Velocity of a flame propagating in the duct versus diameter of the duct L. The noise effect is modeled by a periodic appearance of the N_g new poles at random positions in the rectangle $[0,L]\times[y_{\min},y_{\max}]$. $\gamma=2$, $N_g/L=0.02$, $T=50$.

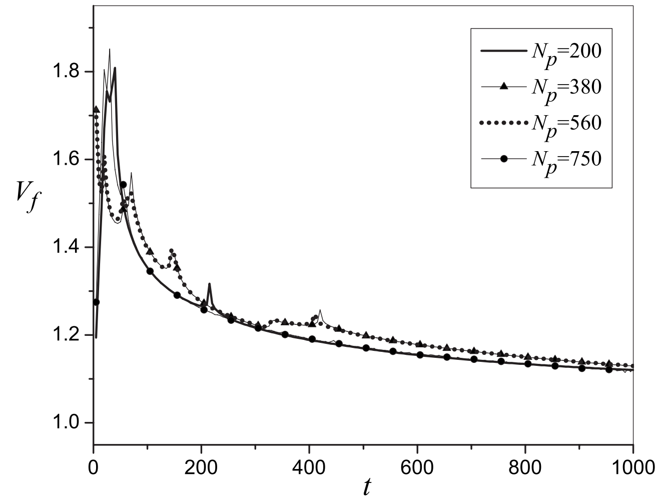

FIG. 4. Time dependency of the outwardly propagating flame velocity calculated by exact pole solutions with a fixed number of poles N_p for $\gamma=2$.

it. The huge amplification of the flame surface area causes a substantial change of the propagation velocity over a finite period of time, although it will decay as $t\to\infty$. This result obtained in terms of an externally perturbed pole solution coincides with the prediction via numerical analysis for the pseudospectra of the linear operator associated with the Sivashinsky equation that is linearized in the neighborhood of the steady solution [20]. The importance of huge transient amplification of perturbations was also found in other physical systems, for example in the Hagen-Poiseuille flow [21]. It is suggestive that under the influence of a periodic flame-front distortion, such strong short-time magnification of small perturbations may result in the formation of a train of secondary cusps that move toward the giant cusp. This dynamics causes the growing of the flame-front surface area and amplification of the propagation velocity.

Numerical calculation for the exact solution of Eq. (1) with periodic ($t_{m+1}-t_m=T=$const) noise term (5) shows a considerable increase of the average propagation velocity in comparison with the velocity given by the noiseless stationary solution. The dependency of flame propagation velocity on the flame size L is shown in Fig. 3. It is assumed that the number of additional poles N_g per unit of the plane flame surface is constant. This physically reasonable assumption was suggested by numerical simulations [20], which show that the number of microcusps on the flame surface grows with the flame size. From the physical point of view, this assumption, for example, may model the propagation of a planar premixed flame through the dusted space with constant dust density. The new poles appeared at random positions inside the rectangle $[0,L]\times[y_{\min},y_{\max}]$ in the complex plane. The real part of the pole position describes the x coordinate of the local maxima on the flame front and the imaginary part characterizes the amplitude of the external perturbation, with a smaller value indicating a larger disturbance amplitude. As can be seen in Fig. 3, the flame front velocity increases with the growth of the flame size, which is in qualitative agreement with the results of numerical simulations [7,13,14,20] and analytical estimations [14]. Thus, unlike the noiseless pole solution, the exact solution of the Sivashinsky equation with force term (5) is capable of capturing the peculiar features of the flame propagation in the planar duct such as dependency of the propagation velocity on the flame size. Based on these results, one may conclude that the increase the flame front velocity with the growth of the flame size is the only effect of the appearance of new poles modeling noises, and the value of velocity amplification depends on the noise intensity.

B. Cylindrical flame

In contrast to the case of flame propagation in a planar duct, in radial condition there is no stable steady state with a finite number of poles. Initially the system can contain infinitely many poles, most of which may have a large negative imaginary part and thus have no contribution to the flame front surface. As was shown in [12], only $N_c=\lfloor R_f(\gamma^2+\sqrt{\gamma^2+8})/4\rfloor$ poles may affect the dynamics of an outwardly propagating flame front of average radius R_f. Imaginary parts of all the other poles increase exponentially with time and, therefore, the corresponding cusps on the flame surface decay exponentially. Yet, since the average radius of the expanding flame increases with time, the value N_c also grows and the imaginary parts of more and more such poles may start to decrease and affect the flame surface significantly and hence the flame propagation velocity. Thus, distinguishing the effects of initial perturbations and external noises on the outwardly propagating flame dynamics is less straightforward [14] than the case of flame propagation in the duct. In order to clarify the behaviors of the pole solutions of the noiseless $[f(\varphi,t)=0]$ Sivashinsky equation (1), which contains an infinitely large number of poles, the set of solutions with a finite but gradually increasing number of poles was considered. Time dependencies of the outwardly propagating flame velocity calculated by the pole solutions with a different number of poles are shown in Fig. 4. As can be seen, the propagation velocity grows only over a short initial stage and subsequently decays to the velocity of an undisturbed flame. Moreover, the expansion rate is almost independent of the number of poles. This fact suggests that such behavior is qualitatively retained even for an infinitely large number of poles in the exact solution of the Sivashinsky equation with zero force term.

The sharp contradictions between the results of direct computation of the Sivashinsky equation [5,7] and analytical results based on the exact solutions (2)–(4) of the noiseless equation (1) are also presented in the case of outwardly propagating cylindrical flames. Therefore, substantial acceleration of the flame front and the fractal-like structure of the expanding flame surface have been observed in numerous numerical simulations but cannot be described in terms of pole solutions conserving the number of poles that exist in the initial conditions [12].

The exact solution of the Sivashinsky equation (1) with impulselike periodic force term (5) was obtained by numerically solving the system of Eq. (4) with a variable number of poles. The N_g new poles appeared with periodicity T at random positions inside the complex rectangle $[0,2\pi]\times[y_{\min},y_{\max}]$. As in the case of a plane flame, we have assumed a constant number of additional poles N_g per unit flame surface. Due to an increase of the mean flame radius, the number of additional poles also increases proportionally to the flame radius. Physically this situation may be associated with sources of the flame perturbations uniformly distributed in the unburned region, for example small particles. When a particle crosses the flame front, it generates a local perturbation that is described by a pole solution.

In the pole expansion, Eq. (2), the harmonics with $n > N_c$, where $N_c=\lfloor R_f(\gamma^2+\sqrt{\gamma^2+8})/4\rfloor$, are exponentially damped [12]. Consequently, the number of equations in the system (4) shall be bounded by the value N_c. Unfortunately, in contrast to the direct simulation of the Sivashinsky equation (1), which can be performed with computational cost of $N_{\text{grid}}\log_2 N_{\text{grid}}$ operations per time step by applying the highly efficient fast-Fourier-transformation technique, the cost of calculating the exact solutions from the system (4) is of order N^2 operations in one time step. This issue presents a significant challenge to computation of N_c equations in the system (4), due to the big value of N_c that is proportional to the flame radius R_f. Since the main purpose of the present work is to qualitatively investigate the noise effect on the pole solutions of Eq. (1), we have restricted ourselves to the calculations of the fewer than N_c equations in the system (4). With the objective to reduce the computing cost, the simulations were conducted in the sector $\varphi\in[0,2\pi/n_{\text{sec}})$ and extended to the whole surface by periodicity. Numerical tests show that the results obtained by calculations in the sector and in the whole circle coincide with each other (Fig. 5).

The time dependences of the flame front velocity evaluated for different values of ratio $P_N=N_g/2\pi R_f$, which represents the number of additional poles per unit flame surface, are presented in Fig. 5. Power-law approximations $(t-t_*)^\alpha$ for the expansion rate of the cylindrical flame are also depicted in Fig. 5. As it is seen, the growth exponent α is almost invariant in the wide range of parameters characterizing the noise intensity, and its value is about 0.75 ± 0.01. Long-term simulations of the Sivashinsky equation (1) show

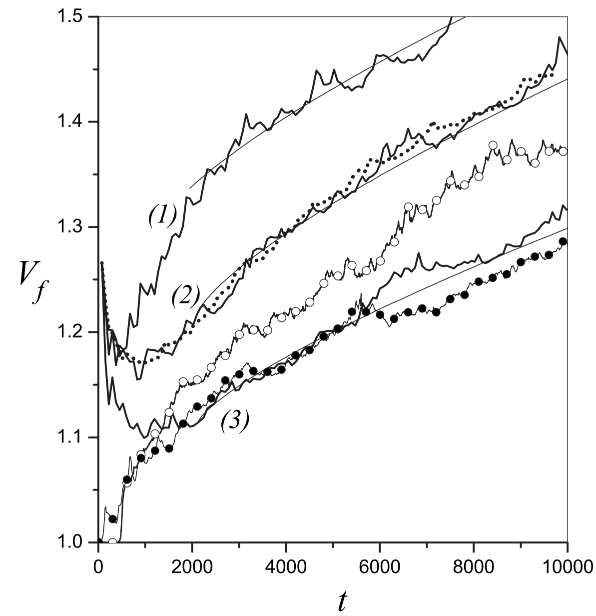

FIG. 5. Time dependency of the outwardly propagating flame velocity calculated by exact pole solutions for $\gamma=2$ and (1) $P_N=0.005$, $T=100$; (2) $P_N=0.002$, $T=100$; (3) $P_N=0.002$, $T=200$. Solid and dashed lines correspond to the calculations in the sector $\varphi\in[0,2\pi/6)$ and in the whole circle, respectively. Marked lines are the results of direct computation of Eq. (1) with zero (solid marks) and random (open marks) force terms.

[7] that the flame expansion rate slows down as the flame size grows and stabilizes to the power law t^α with $\alpha\approx0.25$. The curves marked by solid and open circles in Fig. 5 correspond to the results of direct computation of the Sivashinsky equation (1) with the zero force term and with random periodic noise, respectively. Our estimations of the growth exponent based on the exact solutions of Eq. (1) are in good agreement with the numerical results (marked curves in Fig. 5) obtained for the earlier times. Furthermore, the conclusion of strong correlation between the flame front velocity and the strength of the forcing demonstrated by numerical simulations of Eq. (1) and by exact pole solutions coincides with previous works [7,14]. From the physical point of view, it suggests that upstream velocity perturbations, turbulence, or small particles (for example for a flame propagating through dusted space) may affect the flame front expansion significantly. Hence, the flame expansion rate observed in experiments would depend on testing conditions such as the density of the dust or velocity heterogeneity. Calculation of the pole solutions on the long-time interval, however, meets with difficulty due to the increase of the total number of poles N and the considerable cost of the computational algorithm, which demands N^2 operations per time step. Such long-time simulations are beyond the scope of the present investigation.

Figures 6(a) and 6(b) illustrate the temporal evolution of flame front velocity under noiseless condition [the number of poles N in the system (4) is constant] and in the case of periodic noise but in the absence of hydrodynamic instability ($\gamma=0$), correspondingly. It is shown that the effects of the external forces or hydrodynamic instability alone do not lead

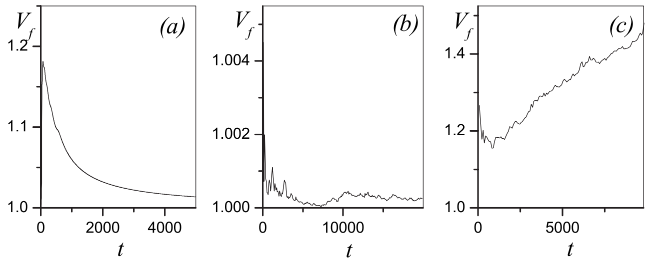

FIG. 6. Temporal evolution of the average flame front velocity calculated for (a) noiseless condition with $\gamma=2$; (b) periodic noise with $T=100$, $P_N=0.002$, and $\gamma=0$; (c) periodic noise with $T=100$, $P_N=0.002$, and $\gamma=2$.

to substantial acceleration of the expanding flame propagation. It can thus be concluded that coupling of the noise effect and intrinsic hydrodynamic instability [Fig. 6(c)] is the essential mechanism of the self-acceleration of the outward propagating flames. Moreover, the calculations testify that an increase of the number of additional poles (i.e., perturbations) N_g with growth of the flame radius is a necessary condition for flame acceleration, whereas the dependency $N_g(R_f)$ does not have to be linear.

Typical shapes of the flame front obtained by calculations of the pole solutions (2)–(4) taking into account the noise generation of new poles are illustrated in Fig. 7. Much as in the case of a plane flame, a new small cusp formed due to noise may move toward a nearby big cusp and merge with it, as indicated by the arrow. But in contrast to the planar geometry, it is not the only fate of a new cusp. The continual growing of the surface area of the outward propagating flame may result in tip splitting when the new secondary cusp remains near the tip between two existing cracks and does not merge with any one of them [15]. As demonstrated in Fig. 7, the pole solutions are capable of describing the cascading structure peculiar to the expanding flame fronts that have been observed in experimental investigations [1] and numerical simulations [7,22,23]. That is the case only when new poles appear in the exact solution (2) and (3) due to the noise effect and, moreover, if the number of additional poles N_g increases with the flame radius.

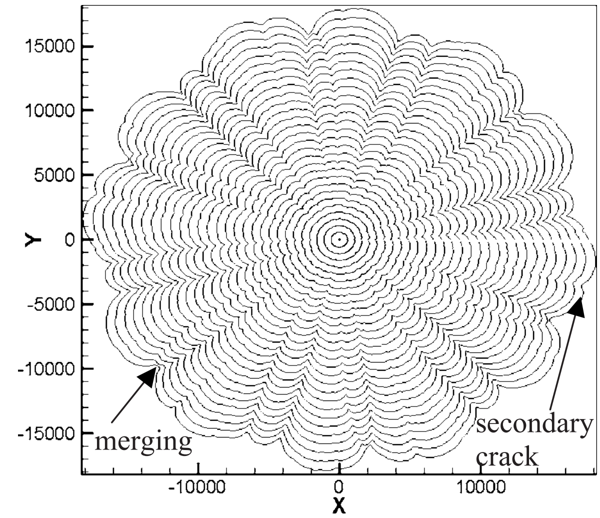

FIG. 7. Evolution of the outwardly propagating cylindrical flame with $\gamma=2$ and $P_N=0.002$.

IV. CONCLUDING REMARKS

The dynamics of hydrodynamically unstable flames has been investigated in terms of the exact solutions of the Sivashinsky equation with a random forcing term modeling an external noise. It is shown that modification of the pole solutions taking into account the appearance of new poles due to the noise captures the features typical for the planar geometry and outwardly propagating flames, which cannot be detected by the pole solutions with a fixed number of poles. Therefore, the increasing of the velocity of the flame propagating in a planar duct with growing of the duct size as well as self-acceleration and cellularization of the expanding flame fronts can be described in terms of pole solutions simulating noises. It is demonstrated that the average velocity of the outwardly propagating flame grows as t^{α}. Although the rate of expansion depends on the forcing strength, the growth exponent α is the same in the wide range of parameters characterizing the noise intensity. These results obtained on the basis of exact pole solutions are quite in line with those of direct computation of the Sivashinsky equation.

Nonsusceptibility of the exact solutions to the roundoff errors allows us to examine the separate influence of the noise and the hydrodynamic instability on the flame front dynamics. It was found that only joint action of the noise and the intrinsic hydrodynamic instability described by the set of pole solutions causes the flame cellularization and substantial self-acceleration. The influence of either of the two factors taken separately does not lead to essential increasing of the flame velocity. This conclusion provides direct support for the idea that the flame acceleration results from explicit and/or implicit forcing, which is always present in the numerical simulation and experimental operation.

ACKNOWLEDGMENTS

The work was supported by the Program for Frontier and Innovative Research of National Taiwan University.

[1] Yu. A. Gostintsev, A. G. Istratov, and Yu. V. Shulenin, Combust., Explos. Shock Waves **24**, 563 (1988).

[2] J. K. Bechtold and M. Matalon, Combust. Flame **67**, 77 (1987).

[3] Y. B. Zeldovich, Combust. Flame **40**, 225 (1981).

[4] G. Sivashinsky, Acta Astronaut. **4**, 1177 (1977).

[5] L. Filyand, G. I. Sivahshinsky, and M. L. Frankel, Physica D **72**, 110 (1994).

[6] Y. D'Angelo, G. Joulin, and G. Boury, Combust. Theory Modell. **4**, 317 (2000).
[7] V. Karlin and G. Sivashinsky, Combust. Theory Modell. **10**, 625 (2006).
[8] M. Rahibe, N. Aubry, G. I. Sivashinsky, and R. Lima, Phys. Rev. E **52**, 3675 (1995).
[9] O. Thual, U. Frisch, and M. Henon, J. Phys. (France) **46**, 1485 (1985).
[10] S. S. Minaev, Combust., Explos. Shock Waves **28**, 30 (1992).
[11] P. Cambray, K. Joulain, and G. Joulin, Combust. Sci. Technol. **103**, 265 (1994).
[12] S. S. Minaev, E. A. Pirogov, and O. V. Sharypov, Combust., Explos. Shock Waves **32**, 481 (1996).
[13] S. Gutman and G. I. Sivashinsky, Physica D **43**, 129 (1990).
[14] Z. Olami, B. Galanti, O. Kupervasser, and I. Procaccia, Phys. Rev. E **55**, 2649 (1997).
[15] O. Kupervasser, Z. Olami, and I. Procaccia, Phys. Rev. Lett. **76**, 146 (1996).
[16] Y. C. Lee and H. H. Chen, Phys. Scr., T **T2**, 41 (1982).
[17] G. Joulin, Zh. Eksp. Teor. Fiz. **100**, 428 (1991).
[18] D. Bessis and J. D. Fournier, J. Phys. (France) Lett. **45**, 833 (1984).
[19] G. Joulin, J. Phys. (France) **50**, 1069 (1989).
[20] V. Karlin, Proc. Combust. Inst. **29**, 1537 (2002).
[21] L. Boberg and U. Brosa, Z. Naturforsch., A: Phys. Sci. **43**, 697 (1988).
[22] K. L. Pan, 45th AIAA Aerospace Sciences Meeting and Exhibit, January, 2007, Reno, NV.
[23] M. A. Liberman, M. F. Ivanov, O. E. Peil, D. M. Valiev, and L.-E. Ericsson, Phys. Fluids **16**, 2476 (2004).